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Base pressure prediction in bluff-body potential-flow models

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In a recent study by Yeung & Parkinson (1997), a wake width was proposed which allowed the bluff-body potential-flow model by Parkinson & Jandali (1970) to be extended to include the flow around an oblique flat plate. By incorporating this wake width in the momentum equation originally derived by Eppler (1954) for separated flow, the drag of the plate is related to its inclination and base pressure through a simple analytical condition. It allows the base pressure, which is usually treated as an empirical input, to be determined theoretically and thus the model becomes self-contained. Predictions of the base pressure, drag and width of wake are found to be in reasonable agreement with the experimental data. When applied to the symmetrical flow around a wedge of arbitrary vertex angle, similar agreement with experimental measurements is obtained as well. It is also demonstrated that this condition is compatible with the free-streamline models by Wu (1962) and Wu & Wang (1964) such that the corresponding predictions are in good agreement with experiment.

1. Introduction

Two distinct regions are generally identified when considering the flow around a bluff body. Upon flow separation, a wake is formed immediately downstream of the body. Low pressure, reversed flow and vortex shedding are some of its characteristics. The region of flow external to the wake is by and large inviscid and potential-flow models for such a region have been reported in the literature.

The earliest potential model for separated flow is the free-streamline theory by Kirchhoff and Helmholtz for the flow normal to a flat plate. The discrepancy between the estimated drag coefficient of 0.88 and the experimental value about 2.0, as indicated in Prandtl & Tietjens (1934), is mainly due to the substantial reduction in pressure behind the plate not modelled in the theory. By specifying the separation velocity, which is linked to the base pressure coefficient, and allowing it to remain constant along the initial portions of the free streamlines until they become parallel to the free stream, the notched hodograph model by Roshko (1954*a*) gives a better estimate of 2.130, as compared to 2.13 measured by Fage & Johansen (1927, hereafter referred to as FJ). Moreover, this potential-flow model by Roshko provides an understanding of the vortex shedding mechanism in the wake. By taking the theoretical wake width and the experimental separation velocity as the characteristic length and velocity respectively, Roshko (1954*b*) proposed a wake Strouhal number, $S^* = 0.164$, which is expected to be universal for all bluff-cylinder wakes. Furthermore, by equating this wake width to the width of the vortex street from von Kármán's formula for the drag, Roshko (1954*b*, 1955) derived a solution dependent only on one experimental measurement, ε, the 'fraction of shear layer vorticity which goes into individual vortices'. As deduced from the theory, the value of base pressure is the same for all cylinders having the same value of ε. With a suitable average value for the base pressure ($C_{pb} = -0.96$) chosen, the calculated drag coefficients and Strouhal numbers for the circular cylinder, 90° wedge and normal flat plate compare well with values obtained experimentally in a range of Reynolds number.

When considering the fully developed wake flow and cavity flow past an inclined flat plate, Wu (1962) generalized Roshko's free-streamline theory by assuming that the complex potential and velocity at the end points of a constant-pressure region bounded by the free streamlines had respectively the same values. The model yields the exact solution in a closed form for the whole range of wake pressure and is applicable to both wake flows in one-phase media and cavity flows in water because the prediction is in remarkably good agreement with experimental observations by FJ and others. Extensions to obstacles of arbitrary profiles such as wedges, flapped hydrofoils and inclined circular arc plates at an arbitrary cavitation number were later reported in Wu & Wang (1964), and the theory is in good agreement with the experimental results. Other independent investigations of the free-streamline theory are found in Mimura (1958), Woods (1961), and Abernathy (1962). Wu (1972) subsequently provided a review of cavity and wake flows with a detailed account on both physical and theoretical aspects. Perspectives on bluff-body aerodynamics, and developments in the understanding of bluff-body flows are recently highlighted in articles by Roshko (1993) and Bearman (1998), respectively.

The wake-source model by Parkinson & Jandali (1970, hereafter referred to as PJ) utilized a different potential-flow approach through the use of conformal transformations and mathematical singularities. In addition to being simpler than the other theories, reliable estimates of drag and pressure distribution upstream of flow separation for two-dimensional bodies including the flat plate normal to the flow, 90°-wedge, and circular and elliptic cylinders have been reported. For example, the pressure distribution on the flat plate is indistinguishable from Roshko's prediction and the calculated drag coefficients agree with measured values within 0.3% for the flat plate and 6.5% for the circular cylinder at subcritical, transcritical and critical Reynolds numbers. The wake-source model subsequently found applications in other studies. For instances, it was adopted in a free-streamline theory for bluff bodies attached to a plane wall by Kiya & Arie (1972) and was incorporated in an analytical model by Güven, Patel & Farell (1977) for high-Reynolds-number flow past rough-walled circular cylinders. Bearman & Fackrell (1975) developed a numerical method incorporating some of the ideas of PJ to calculate the potential flow external to two-dimensional and axisymmetric bluff bodies. A version of the wake-source model to include the far-wake displacement, originally proposed by Woods (1961), was reported by Kiya & Arie (1977). Recently, an analytical expression for the wake width, which was suitably deduced from the physical evidence based on well-documented experimental data from Abernathy but without additional empirical parameters, was proposed by Yeung & Parkinson (1997, hereafter referred to as YP) to extend the wake-source model to steady separated flow around an inclined flat plate. Based on such a wake width and the separation velocity deduced from the base pressure measurements by Abernathy, the modified Strouhal number is found to be independent of the inclination.

The success of the above-mentioned inviscid models relies on the base pressure, which is always specified experimentally because the flow in the wake is not modelled. The purpose of this study is to demonstrate how this empirical input might be eliminated such that these potential-flow models become self-contained. It is found that if the wake width proposed in YP is utilized in a momentum equation for bluff bodies originally proposed by Eppler (1954), then the drag is related to the base pressure and the inclination of the plate. Independently, the wake-source model can be used to provide another relationship among the drag, base pressure and inclination. As a result, the base pressure is simply found by solving simultaneously a system of nonlinear algebraic equations at any particular inclination. Results show that the predicted base pressure, drag and wake width agree reasonably well with the experimental data of FJ for the inclined flat plate and the measurements of Simmons (1977) for the wedge of arbitrary vertex angle. This relationship, when incorporated in the free-streamline models by Wu (1962) and Wu & Wang (1964), produces similar good agreements.

2. Flat plate

Following the formulation in YP for the flow around an inclined flat plate of length c, the complex velocity in the transform plane ζ, which contains a unit circle centred at the origin, is given by

$$W(\zeta) = V\left\{1 - \frac{1}{\zeta^2} + \frac{\mathrm{i}\Gamma}{2\pi V\zeta} + \frac{Q_1}{\pi V(\zeta - \exp[\mathrm{i}\delta_1])} + \frac{Q_2}{\pi V(\zeta - \exp[\mathrm{i}\delta_2])} - \frac{Q_1+Q_2}{2\pi V\zeta}\right\}, \tag{1}$$

where V is the free-stream speed in the ζ-plane. $2Q_1$ and $2Q_2$ are the two surface sources added to the circle at angular locations δ_1 and δ_2 to create two stagnation points, which are 180° apart on the circumference, coinciding with the leading and trailing edges of the plate in the z-plane where $z = x + \mathrm{i}y$. Γ is the strength of a vortex added to the centre of the circle to provide non-zero lift around the flat plate. The inclination α with respect to the free stream of speed U is related to V by

$$U \exp(-\mathrm{i}\alpha) = V\left.\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|_{\infty}, \tag{2}$$

where

$$\frac{\mathrm{d}z}{\mathrm{d}\zeta} = \frac{c}{4}\left(\mathrm{e}^{\mathrm{i}\alpha} - \frac{1}{\mathrm{e}^{\mathrm{i}\alpha}\zeta^2}\right). \tag{3}$$

When the values of Γ, Q_1, δ_1, Q_2 and δ_2 are known, the pressure distribution on the upstream surface of the plate is determined through Bernoulli's equation,

$$C_p = \frac{2(p-p_\infty)}{\rho U^2} = 1 - \left|\frac{W(\zeta)}{U}\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|^2, \tag{4}$$

from which the sectional drag F_D on the plate in dimensionless form is found by integration,

$$C_d = \frac{2F_D}{\rho U^2 c} = \frac{\sin\alpha}{c}\int_0^c (C_p - C_{pb\alpha})\,\mathrm{d}y, \tag{5}$$

where $C_{pb\alpha}$ is the base pressure at inclination α.

The local boundary conditions applied to the leading edge, $\zeta = \exp(\mathrm{i}[\pi - \alpha])$, and

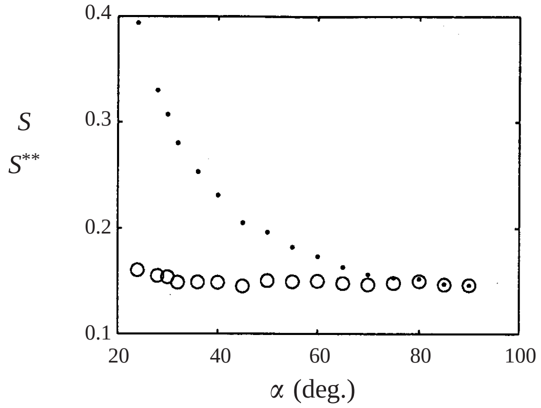

FIGURE 1. Variations of Strouhal numbers with flat-plate inclination α from FJ. ●, S; ○, S^{**}.

trailing edge, $\zeta = \exp(-\mathrm{i}\alpha)$, are

$$W(\zeta) = 0, \tag{6}$$

and

$$C_p(z) = C_{pb\alpha}, \tag{7}$$

where $z = 0$ and c. Equations (6) and (7) provide a total of four conditions.

The base pressure behind the plate is associated with the downstream periodic vortex shedding. In the theory of YP, it was incorporated into the wake width measured in the direction normal to the uniform flow far upstream as

$$D^* = \sqrt{1 - C_{pb\alpha}}\, c\, g(\alpha), \tag{8}$$

where $g(\alpha) = \sin\alpha$. D^* was proposed after examining the experimental data of Abernathy (1962). As indicated in figure 2 of YP, the modified Strouhal number based on D^*,

$$S^{**} = \frac{n D^*}{\sqrt{1 - C_{pb\alpha}}\, U}, \tag{9}$$

where n is the frequency of vortex shedding, is identical to the conventional Strouhal number based on projected plate width and is nearly a constant within the range $30° \leqslant \alpha \leqslant 90°$. Further independent evidence in demonstrating that D^* is the appropriate wake width is depicted in figure 1, where the measurements of FJ are used to compare the variations of $S = nc/U$ and S^{**} over $24° \leqslant \alpha \leqslant 90°$. Based on the continuity equation and D^*, the fifth boundary condition evolved is

$$\frac{Q_1 + Q_2}{2Q} = \frac{k_\alpha \sin\alpha}{k_{90°}}, \tag{10}$$

where $k_\alpha = \sqrt{1 - C_{pb\alpha}}$.

To eliminate the empirical input of base pressure, an additional boundary condition is needed. By the momentum equation, the drag coefficient of a symmetrical body was given by Eppler as

$$C_d = -C_{pb\alpha}\frac{D}{c}, \tag{11}$$

where D is the wake width also measured in the direction perpendicular to the

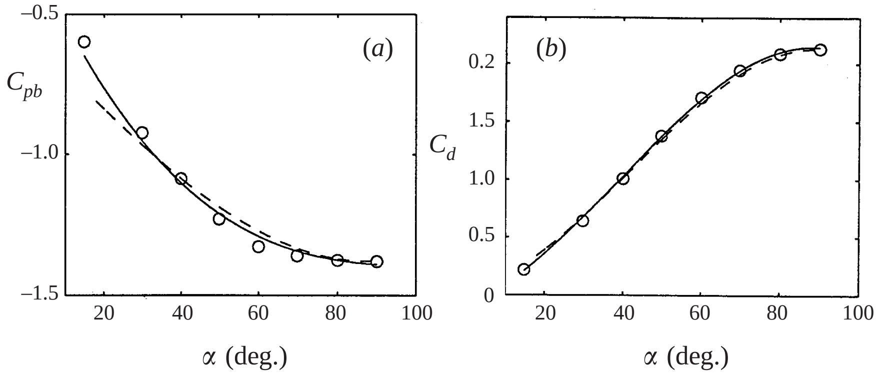

FIGURE 2. Variations of (*a*) base pressure and (*b*) drag with α for inclined flat plate. ○, FJ; ——, present work; – – –, Wu (1962).

upstream flow. Setting $D = D^*$, (11) becomes

$$C_d = (k_\alpha^2 - 1)k_\alpha \sin\alpha. \tag{12}$$

Therefore, the values of $\Gamma, Q_1, \delta_1, Q_2, \delta_2$ and $C_{pb\alpha}$ are obtained by solving (6), (7), (10) and (12) simultaneously with the drag coefficient provided by (5).

The uniform flow past a normal flat plate was considered in PJ, and with $\alpha = 90°$ (10) is reduced to an identity, providing no additional information on the unknowns. By symmetry, however, $\Gamma = 0$, $Q_1 = Q_2 = Q$ and $\delta_1 = -\delta_2 = \delta$, where the source strength and location are, respectively

$$Q = \tfrac{1}{2}\pi U c \cos\delta, \qquad \sec\delta = k_\alpha. \tag{13}$$

The drag coefficient derived in PJ is

$$C_d = 3 - \pi\cos\delta + \frac{\cos 2\delta}{\sin\delta}\ln\left[\frac{1+\cos\delta+\sin\delta}{1+\cos\delta-\sin\delta}\right] + \tan^2\delta. \tag{14}$$

Equations (12), (13) and (14) are solved simultaneously to give $C_{pb\alpha} = -1.385$ and $C_d = 2.139$, compared to the experimental values $C_{pb\alpha} = -1.38$ and $C_d = 2.13$ from FJ. The corresponding pressure distribution is indistinguishably close to the results reported in figure 3 of PJ.

The above theoretical values of $C_{pb\alpha}$ and Q for $\alpha = 90°$ may be used in (10) when considering cases where $\alpha < 90°$. The lack of symmetry, however, requires Q_1, Q_2, δ_1, δ_2, Γ and $C_{pb\alpha}$ to be determined simultaneously and iteratively by solving transcendental equations (6), (7), (10) and (12), which may involve the numerical integration to obtain C_d by using (5). The predicted values of $C_{pb\alpha}$ and C_d with respect to α are shown in figures 2(*a*) and 2(*b*) respectively, and agree reasonably well with the experimental data from FJ. A discussion on the validity of FJ's data is provided in a later section. The corresponding variation of wake width D/c with α is depicted in figure 3, which also includes the measurements of 'separation between free-vortex layers' for the case of minimum blockage by Abernathy and the wake width calculated by FJ, using von Kármán's stability relation and the measured values of the vortex spacing. Figure 4 indicates that at given values of α, $C_d/\sin\alpha$ is a simple cubic function of k_α, as expressed in (12). Because of the close agreement between the experimental and theoretical values of $C_{pb\alpha}$, the predicted pressure distributions at $\alpha = 69.85°, 49.85°, 29.85°, 14.85°$ (i.e. figures 3 to 6 in YP) remain unchanged.

Using the free-streamline theory, the drag coefficient for the fully developed wake

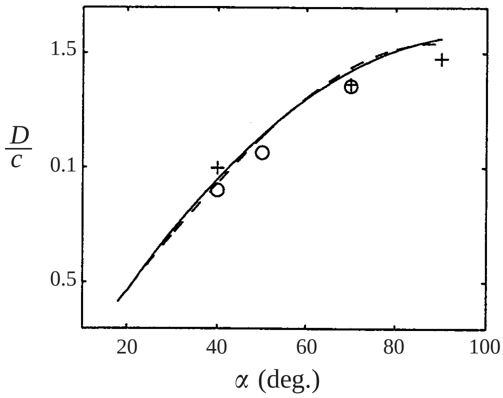

FIGURE 3. Variations of wake width with α for inclined flat plate. ○, Abernathy (1964); +, FJ; ——, present work; – – –, Wu (1962).

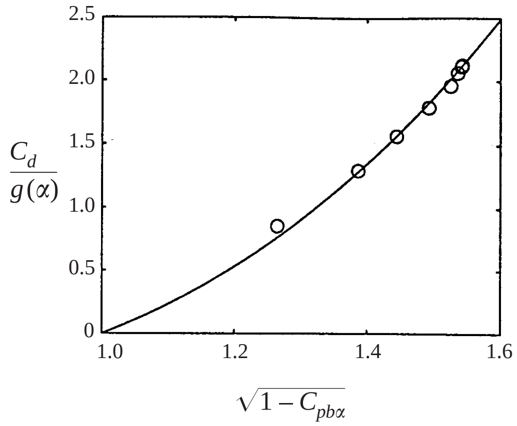

FIGURE 4. Variations of $C_d/g(\alpha)$ with base pressure for inclined flat plate. ○, FJ; ——, present work.

flows and cavity flows past an oblique flat plate was derived by Wu (1962) and is almost identical to that of (14) as shown in figure 5 for $\alpha = 90^\circ$ and $-2 < C_{pb} < 0$. The intersection at about $C_{pb\alpha} = -1.385$ and $C_d = 2.139$ was obtained when (12) was incorporated in Wu's result. At any other inclination, the theoretical variation of $C_{pb\alpha}$ can be found iteratively and is shown in figure 2(*a*), while the predicted drag coefficient is included in figure 2(*b*). The corresponding wake width, which is also related to the drag coefficient and base pressure through (11), is shown in figure 3. In general, the variations of $C_{pb\alpha}$, C_d and wake width are in good agreement with the present model and data from FJ and Abernathy. Two other points of intersection are found in figure 5 near $(C_{pb}, C_d) = (-0.96, 1.74)$ and $(-1.051, 1.503)$, corresponding to solutions obtained by matching von Kármán's drag coefficient (see Roshko 1954*b*, 1955 where the width of the vortex street is equated to D^*) with (14) and (12), respectively. In brief, Eppler's drag equation together with D^* is more realistic in predicting the base pressure and drag. It should be noted that using the free-streamline theory, Wu independently derives an expression similar to (11) for a flat plate at an arbitrary inclination. Therefore, although (11) was originally obtained by Eppler for a symmetrical body, its extension to non-symmetrical bodies is justified.

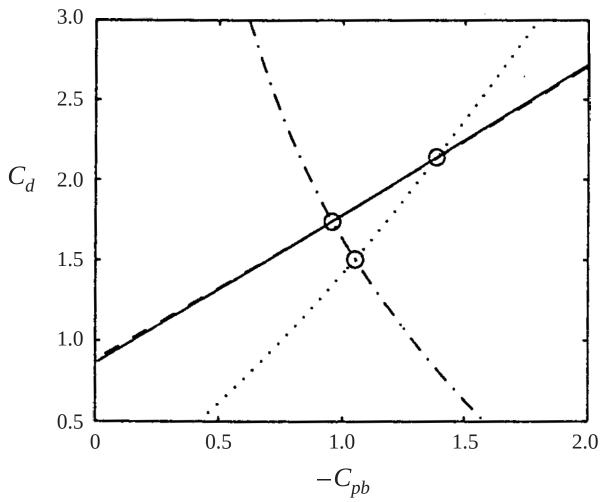

FIGURE 5. Comparison of theoretical drag for normal flat plate. ——, PJ; – – –, Wu (1962); –·–·–, Kármán; ·····, present work.

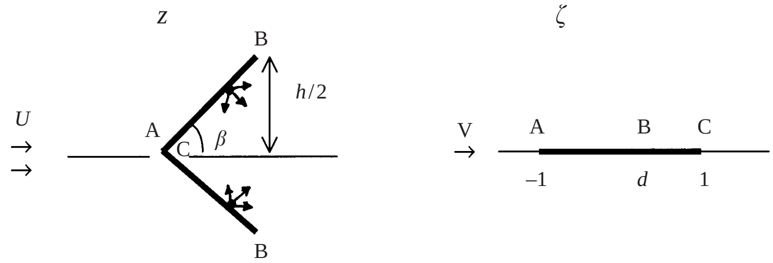

FIGURE 6. Conformal mapping for symmetrical wedge of angle 2β.

3. Wedges

The symmetric flow around a 90°-wedge was also considered by PJ. The wake-source model is easily extended to include a wedge of an arbitrary vertex angle 2β as shown in figure 6 with h being the base height, when utilizing the Schwarz–Christoffel transformation

$$\frac{\mathrm{d}z}{\mathrm{d}\zeta} = m\left(\frac{\zeta-1}{\zeta+1}\right)^{\beta/\pi}\frac{\zeta-d}{\zeta-1}. \tag{15}$$

The upper half-ζ-plane, cut along ABC where $\zeta_A = -1, \zeta_B = d = 1 - 2\beta/\pi$ and $\zeta_C = 1$, is mapped onto the upper z-plane with $z_A = 0$ and $z_C = 0$. Upon integration with point B located at $z_B = h(\cot\beta + i)/2$, (15) becomes

$$z = m(\zeta+1)^a(\zeta-1)^b, \tag{16}$$

where $a = 1 - b$, $b = \beta/\pi$ and $m = h\,a^{-a}\,b^{-b}/(4\sin\beta)$. At $2\beta = 90°$, (15) is identical to equation (5.1) in PJ, while (16) is much simpler than their (5.2). Following the formulation in PJ, the strength and location of the double source are then given by

$$Q = \pi m U(\varepsilon - d), \tag{17}$$

$$\varepsilon = d + \frac{m}{pk_\beta}, \tag{18}$$

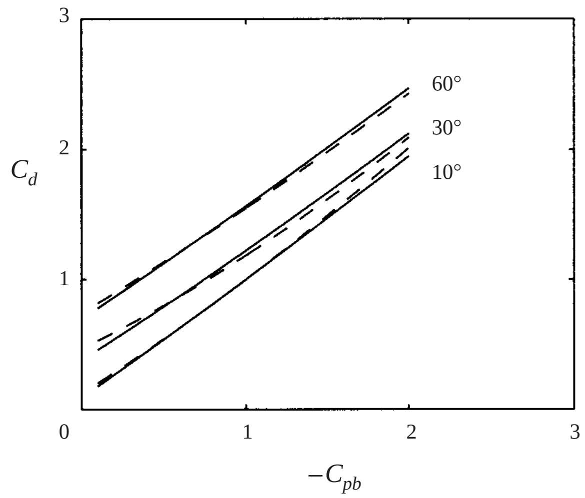

FIGURE 7. Comparison of theoretical drag for wedge of angle 2β. ——, Present work; – – –, Wu & Wang (1964).

where $p = |\mathrm{d}^2z/\mathrm{d}\zeta^2|$ at $\zeta = d$ and $k_\beta = \sqrt{1 - C_{pb\beta}}$. The pressure distribution on the wetted surface of the wedge is

$$C_p = 1 - \left| \left(1 + \frac{m}{pk_\beta(\zeta - \varepsilon)} \right) \frac{\mathrm{d}\zeta}{\mathrm{d}z} \right|^2, \tag{19}$$

and the drag coefficient is given by

$$C_d = 2 \int_0^{h/2} (C_p - C_{pb\beta})\, \mathrm{d}y. \tag{20}$$

In figure 7, the variations of C_d from the present model over a wide range of base pressure at $\beta = 10^\circ, 30^\circ$ and 60° are quite close to those from Wu & Wang, which have been found to be in remarkable agreement with the experimental results of Waid (1957) and others for cavity flow. To eliminate the empirical input of base pressure, the theoretical wake with D^*, not necessarily the same as (8) because of the change of flow structure, is to be determined.

Based on a universal Strouhal number devised by Calvert (1967) on axisymmetric bodies, Simmons (1977) found that the modified Strouhal number defined as

$$S_{CA} = \frac{n\, l_W}{\sqrt{1 - C_{pb\beta}}\, U}, \tag{21}$$

where l_W is 'the distance apart of the root mean square peaks in velocity found in a traverse made at right angles across the wake in line with the point of minimum pressure on the wake centre line', remained a constant for a number of two-dimensional wedge-shaped bodies with the boundary-layer separation angle at $\beta = 0^\circ, 10^\circ, 15^\circ, 20^\circ, 30^\circ, 60^\circ$ and 90°. Similar to the flow around an inclined flat plate, (21) strongly suggests that l_W of the wedge is similar to D^* of the flat plate. It is, therefore, assumed that for the wedge

$$D^* = \sqrt{1 - C_{pb\beta}}\, h\, g(\beta), \tag{22}$$

where $g(\beta)$ may be derived by studying S_{CA} and $S = nh/U$. By comparing the

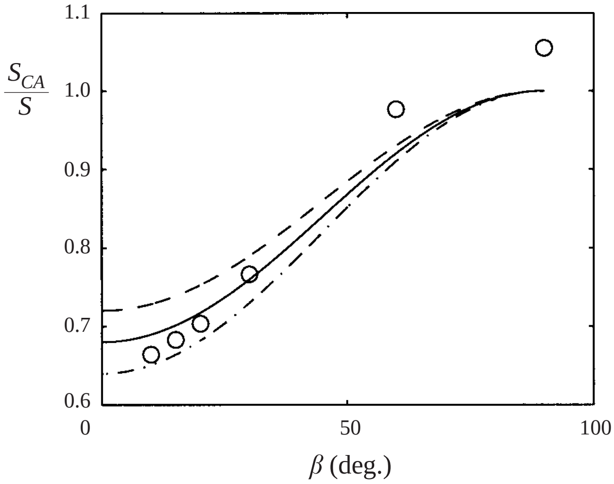

FIGURE 8. Variations of Strouhal number ratio with β. ○, Simmons (1977); – – –, $B = 0.14$; ——, $B = 0.16$; –·–·–, $B = 0.18$.

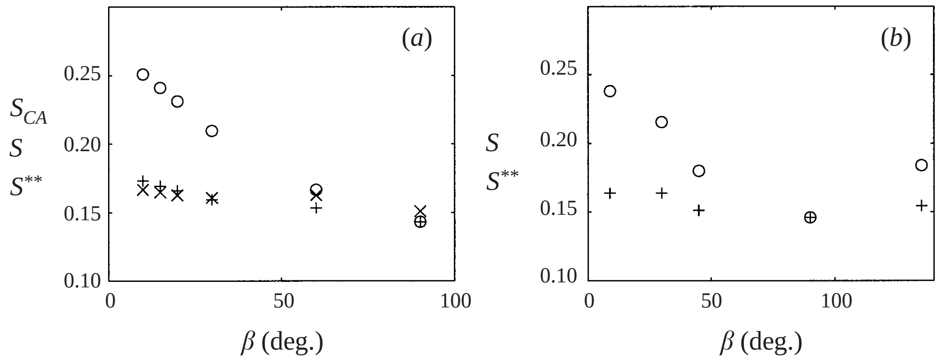

FIGURE 9. Comparison of Strouhal numbers. (*a*) ○, S; ×, S_{CA} from Simmons (1977); +, S^{**} present work. (*b*) ○, S from various sources; +, S^{**} present work.

orientations of the separation streamlines of the inclined flat plate and of the wedge, $g(\beta)$ may take the general form

$$g(\beta) = A + B\sin(C\beta + D), \tag{23}$$

where A, B, C and D are constants. By symmetry, the flow structure at $0 < \beta < 90°$ and $90° < \beta < 180°$ are repeated at $-90° < \beta < 0$ and $-180° < \beta < -90°$, respectively. Therefore, $g(\beta) = g(\beta + 2\pi)$ is deduced and it leads to the requirement that $C = \pm 1, \pm 2, \dots$. From the variation of S_{CA}/S with β in figure 8, $dg/d\beta \approx 0$ as $\beta \to 0$ and 90°. Therefore, $C = 2$ and $D = \pi/2$, if the smallest positive value of C is chosen. In addition, $g(\beta = 90°)$ should be identical to $g(\alpha = 90°)$. When $\beta = 90°$, Simmons' data however indicate that $g(\beta) \approx 1.055$, which is slightly higher than $g(\alpha) = 1$ for the flat plate at $\alpha = 90°$. To be consistent with the results of the flat plate, $g(\beta) = 1$ at $\beta = 90°$ is adopted and it leads to

$$g(\beta) = 1 - B - B\sin(2\beta + \tfrac{1}{2}\pi). \tag{24}$$

The variation of $g(\beta)$ for a few values of B as depicted in figure 8 suggests that

$B = 0.16$ is an appropriate choice. In figure 9(a), both S_{CA} and $S^{**} = nD^*/k_\beta U = S\,g(\beta)$ are compared with S. Further comparisons are made in figure 9(b) where values of S from Fage & Johansen (1928) for $\beta = 8.9°$ and 90°, Twigge-Molecey & Baines (1973) for $\beta = 30°$, Roshko (1954b) for $\beta = 45°$ and Belvins (1984) for $\beta = 135°$ are plotted with S^{**}, demonstrating that S^{**} is reasonably constant for a wide range of apex angle.

By equating the wake width to D^*, the drag coefficient defined in (11) becomes

$$C_d = (k_\beta^2 - 1)k_\beta\, g(\beta), \tag{25}$$

and it can be combined with (20) to determine $C_{pb\beta}$. Note that $\beta = 0°$ corresponds to $h = 0$ in figure 6 so that this case is not equivalent to that given in Simmons (1975) and is excluded in the following discussion. The theoretical prediction of $C_{pb\beta}$ in figure 10(a) is in reasonable agreement with data from Simmons (1977) and Twigge-Molecey & Baines but is quite different from the measurements by Waid ($\beta = 5°, 15°, 45°$ and 90°), and by Roshko (1954b, $\beta = 45°$). However, the base pressure for $\beta = 135°$ in figure 11 from Belvins is unavailable for comparison. Figures 11(a) to 11(d) show the predicted pressure distributions at $\beta = 10°, 15°, 20°$ and 60° when the theoretical base pressure is used in each case. Additionally, the pressure measurements by Simmons (1975) and Twigge-Molecey & Baines are compared with the theoretical prediction at $\beta = 30°$, as depicted in figure 11(e). The values of C_d obtained from (25) are shown in figure 10(c) with the data from Waid, Simmons (1977), Lindsey (1938) for $\beta = 15°, 30°, 45°$ and 60°, Roshko ($\beta = 45°$) and FJ ($\beta = 90°$). The variation of the theoretical wake width D^* is in reasonable agreement with l_W from Simmons (1975), as shown in figure 10(b). Interestingly, the experimental data from Simmons (1977), Twigge-Molecey & Baines, Roshko and FJ collapse onto a simple cubic function between $C_d/g(\beta)$ and k_β from (25) in figure 12, but the data from Waid show another trend, suggesting that (25) is suitable for wake flows.

Relation (25) may be readily combined with the free-streamline model by Wu & Wang where the relationship among C_d, base pressure and vertex angle is available. The resulting theoretical variations of $C_{pb\beta}$, C_d and wake width deduced from (11) are also plotted in figure 10. Similar to the flat plate, the predictions from the free-streamline model are quite close to the present model.

4. Discussion

As indicated in an added note in FJ, the experimental data for the inclined flat plate should be corrected for the interference of the wind tunnel. With the blockage ratio of 1/14 (or 7.2%), the measured values of drag should be reduced by amounts varying from 13.5% at $\alpha = 90°$ to 8% at $\alpha = 30°$, as suggested by FJ. Nevertheless, the 'uncorrected' FJ data have been extensively used for comparison with theory in Roshko (1954), Mimura (1958), Abernathy (1962), Wu (1962), PJ, Kiya & Arie (1977), Bearman & Fackrell (1975), and YP. Perhaps it is because the base pressure is an empirical input. Recently, the uncorrected data were, however, considered by Roshko (1993) as useful input to the numerical simulations by Chua *et al.* (1990) in which the calculated base pressure was found to exhibit suction lobes. We should address the wind tunnel corrections of FJ's data because deteriorating agreement between the present theory and experimental might result, if corrections are made.

As noted by Abernathy, his measurements of base pressure for the flat plate were 'fairly poor' in comparison with data from FJ for the same blockage ratio. He attributed it to the effect of the clearance between the plate and the tunnel walls

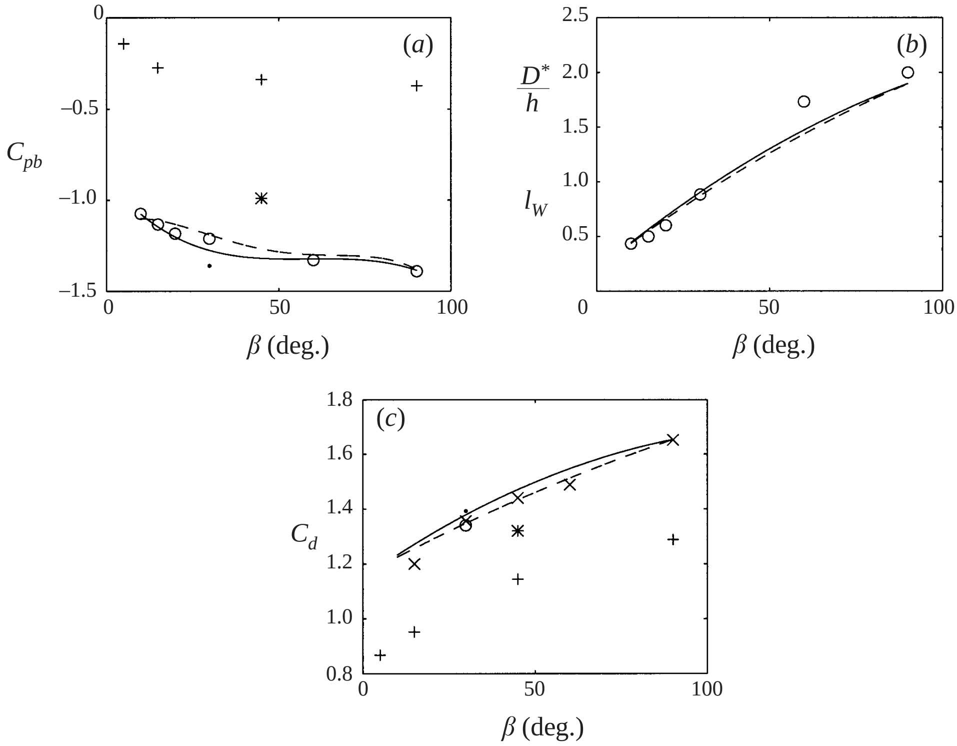

FIGURE 10. Variations of (*a*) base pressure, (*b*) wake width and (*c*) drag with β. ●, Twigge-Molecey & Baines (1973); +, Waid (1957); ○, Simmons (1977); ∗, Roshko (1954*b*); ×, Lindsey (1938); ——, present work; – – –, Wu & Wang (1964).

in FJ's experiments. By using the drag coefficient by Flachsbart (1932), Abernathy corrected FJ's value of base pressure at $\alpha = 90°$ for end leakage and the resulting $C_{pb} = -1.55$ agrees well with his value at -1.50 (about 3% in difference). Following Abernathy's procedure, FJ's pressure distribution on the plate may be corrected for leakage and a drag coefficient of 2.27 is obtained by integrating the resulting pressure coefficient. To correct for the blockage effects, the well-known method of Allen & Vincenti (1944) may be used. The pertinent formulas, as quoted by Roshko (1961) for flow past a circular cylinder at high Reynolds number, are

$$C_p = 1 + \left(\frac{V'}{V}\right)^2 (C_p' - 1), \tag{26}$$

$$\frac{V}{V'} = 1 + \frac{C_d'}{4}\left(\frac{d}{h}\right) + 0.82\left(\frac{d}{h}\right)^2, \tag{27}$$

$$\frac{C_d}{C_d'} = 1 - \frac{C_d'}{2}\left(\frac{d}{h}\right) - 2.5\left(\frac{d}{h}\right)^2, \tag{28}$$

where d/h is the blockage ratio, V, C_p and C_d are the corrected values of velocity, pressure and drag coefficients, and V', C_p' and C_d' are the corresponding measured values. Taking $d/h = 1/14$, $C_d' = 2.27$ and $C_p' = -1.55$, it was found that $C_p = -1.33$ and $C_d = 2.06$, about 4% and 3% different from the respective uncorrected data from

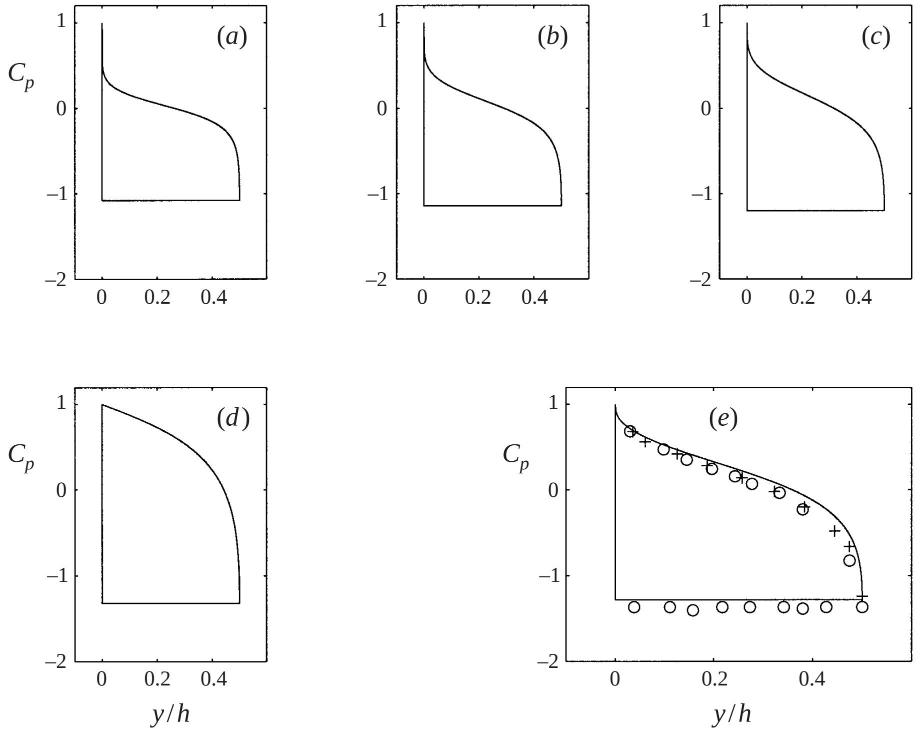

FIGURE 11. Pressure distributions on wedges. (*a*) $\beta = 10°$; (*b*) $\beta = 15°$; (*c*) $\beta = 20°$; (*d*) $\beta = 60°$; (*e*) $\beta = 30°$. +, Simmons (1975); ∘, Twigge-Molecey & Baines (1973).

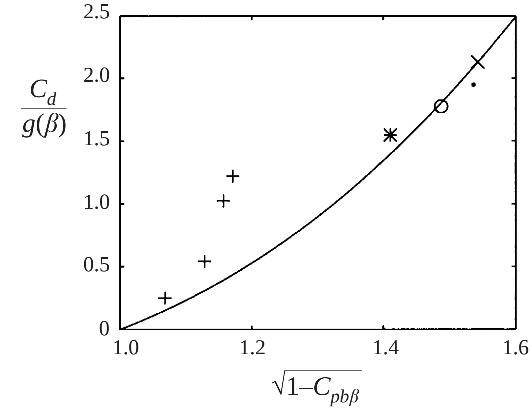

FIGURE 12. Variations of $C_d/g(\beta)$ with base pressure. ●, Twigge-Molecey & Baines (1973); ○, Simmons (1977); ∗, Roshko (1954*b*); +, Waid (1957); ×, FJ; ——, present work.

FJ. From an independent investigation by Simmons (1977) with a blockage ratio of 3.5%, the base pressure coefficient was about −1.37, which is quite close to the uncorrected and corrected values of FJ. As a result, the uncorrected measurements of FJ's data are suitable for comparison with theory, even though the base pressure is not an empirical input.

It is interesting to note that the mean values of the modified Strouhal number S^{**} based on the proposed wake width D^* are of the same order of magnitude, namely

0.15 from figure 1, 0.164 (see figure 2 of YP), 0.161 from figure 9(*a*) and 0.156 figure 9(*b*). In addition, they are, incidentally, close to the values of the universal Strouhal number $S^* = 0.164$ by Roshko (1954*b*), and $S_{CA} = 0.163$ by Simmons (1977), among others. It is, however, not the intention here to define another universal Strouhal number, which should be independent of Reynolds number and geometry.

The present study suggests that the proposed wake width, which is related to the separation velocity, sufficiently represents the mean properties of wake dynamics and thus provides a link between the wake and the separation condition because the resulting modified Strouhal number is independent of the change of flow structure. On the other hand, the momentum equation for separated flow by Eppler combines the drag from reliable potential flow models with such a wake width to produce realistic predictions of the base pressure, drag and wake width. It is of interest to extend the present work to two-dimensional bodies of continuous curvature over which flow separation is induced by boundary layers, such as a circular cylinder. Williamson (1996) recently presented a detailed overview on the vortex dynamics phenomena in the wake of a circular cylinder, over a wide range of Reynolds numbers. Extensive experimental results concerning the variations of base pressure (e.g. Roshko 1993), Strouhal number (e.g. Bearman 1969; Schewe 1983; Williamson & Roshko 1990) and drag (e.g. Roshko 1961; Schewe 1983) with respect to Reynolds number are available. However, detail and consistent measurements of the angle of flow separation are limited (e.g. Achenbach 1968; James, Paris & Malcolm 1980). Furthermore, the presence of separation bubbles, effects of surface roughness and the three-dimensional aspects of nominally two-dimensional wake flows may provide more complications in linking the wake and the separation condition.

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